Tools for Formal Semantics

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Announcements

- By 11:59pm today
  - Fill out the poll for student hours
- For next Wednesday
  - Personal Learning Goals Part 1 due
  - Read van Eijck and Unger Chapter 3
  - (Try to) install Haskell
    - `https://www.haskell.org/ghcup/`

Today's Plan

- Course Plan
- Tools for Formal Semantics
  - Sets
  - Relations
  - Functions
  - Lambda Calculus
  - Types

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  - (we'll see how far we get...)

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  - Tools for building models: sets, relations, functions, types
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  - Functional programming in Haskell as an implementation of formal semantics

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  - Tools for building models: sets, relations, functions, types
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  - Propositional and predicate (first-order) logic for evaluating the truth of sentences
  - Functional programming in Haskell as an implementation of formal semantics
- We will be able to translate expressions (from a fragment of English) into a logical form

Course Plan

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    - Intensional constructs, attitude verbs, time and tense, etc.

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  - Intensional constructs, attitude verbs, time and tense, etc.
- We will introduce possible worlds to model these expressions
- Functors (specifically applicative functors) allow us to compose meanings within a possible world
- Modal logic allows us to evaluate the truth of sentences across possible worlds

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  - Resolving scope ambiguity, anaphora, etc.

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- Continuations (a type of monad) allow us treat expression meanings as functions of their contexts
- These can be applied to both sentence-level and discourse-level contexts

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  - If we have time:  , 

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- Other topics
  - Meaning in languages other than English
  - Semantics of non-declarative sentences (e.g., interrogatives, imperatives, etc.)
  - Distributional semantics of expressions other than words
  - Computational lexical semantics
  - Meaning representations (other than logical forms or vectors)
  - etc.

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- This is where your paper presentations and (if you're up for it) final project come in

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- Principle of extensionality: if two sets have the same elements, then they are equal
  - In other words, "sets are fully determined by their members"

Sets

- Ways to specify a set
  - List its members
    - $A = \{1, 2, 3\}$
    - Order and multiplicity don't matter: $\{\text{red}, \text{white}, \text{blue}\} = \{\text{white}, \text{blue}, \text{white}, \text{red}\}$
  - Describe it in language
    - "the set of colours of the Dutch flag"
  - Set comprehension
    - $E = \{2n | n \in \mathbb{N}\}$

Sets

- Special sets
  - $\mathbb{N}$: the set of natural numbers
  - $\mathbb{Z}$: the set of integers
  - $\varnothing$: the empty set

Sets

- If every element of a set A is also an element of a set B, then A is a subset of B: $A \subseteq B$
- $A = B$ iff (if and only if) $A \subseteq B$ and $B \subseteq A$

Sets

- Set operations
  - Union: $A \cup B$ is the set of objects that are either elements of A or elements of B
  - Intersection: $A \cap B$ is the set of objects that are both elements of A and elements of B
  - Complement: $\overline{A}$ is the set of objects in the universe (or domain) U that are not elements of A
  - Difference: $A - B$ is the set of objects that are elements of A but not elements of B
  - Cartesian product: $A \times B$ is the set of all ordered pairs (a, b) such that a is an element of A and b is an element of B

Sets

- Exercises from the book
  - (I encourage you all, as you are reading, to do the exercises, or at least figure out how you could do them.
  - Some of the easier exercises might show up in class; some of the harder exercises might show up on your homework...)

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  - **Exercise 2.1** Explain why $\varnothing \subseteq A$ holds for every set A.
  - **Exercise 2.2** Explain the difference between $\varnothing$ and $\{\varnothing\}$.

Sets

- Exercise not from the book
  - Let $A = \{1, 2\}$, $B = \{2, 3\}$, and $U = \{0, 1, 2, 3\}$. What are...
    - $A \cup B$
    - $A \cap B$
    - $\overline{A}$
    - $A - B$
    - $A \times B$

Relations

- A relation between two sets A and B is a set of ordered pairs (a, b) such that $a \in A$ and $b \in B$
  - A relation between A and B is a subset of $A \times B$

Relations

- More generally, we can have relations between more than two sets
  - A binary relation is a set of ordered pairs
  - A ternary relation is a set of ordered triples
  - ...
  - An n-ary relation is a set of n-tuples (ordered sequences of n objects)

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  - An n-ary relation is a set of n-tuples (ordered sequences of n objects)
- More generally, we can have relations between fewer than two sets
  - A unary relation (property) of A is a subset of A

Relations

- Operations on relations
  - Let R and S be binary relations on a set U (i.e., between U and itself)
  - Composition: $R \circ S$ is the set of pairs (x, y) such that there is some z with $(x, z) \in R$ and $(z, y) \in S$
  - Converse: $R^{\smile}$ is the set of pairs (y, x) such that $(x, y) \in R$

Relations

- Properties of relations
  - R is reflexive iff for all $x \in U$, $(x, x) \in R$
  - R is symmetric iff for all $x, y \in U$, if $(x, y) \in R$, then $(y, x) \in R$
  - R is transitive iff for all $x, y, z \in U$, if $(x, y) \in R$ and $(y, z) \in R$, then $(x, z) \in R$

Relations

- Exercises from the book
  - **Exercise 2.5** What is the composition of $\{(n, n+2) | n \in N\}$ with itself?
  - **Exercise 2.9** Can you give an example of a transitive relation R for which $R \circ R = R$ does not hold?

Functions

- "Functions are relations with the following special property: for any (a, b) and (a, c) in the relation it has to hold that b and c are equal.
- Thus a function from a set A (called domain) to a set B (called range) is a relation between A and B such that for each $a \in A$ there is one and only one associated $b \in B$.
- In other words, a function is a mechanism that maps an input value to a uniquely determined output value."

Functions

- Extensional view of functions: functions as ordered pairs

Kelvin	Celsius	Fahrenheit	
0	−273.15	−459.67	(absolute zero)
273.15	0	32	(freezing point of water)
310.15	37	98.6	(human body temperature)
373.13	99.98	211.96	(boiling point of water)
505.9	232.8	451	(paper auto-ignites)

Functions

- Intensional view of functions: functions as instructions for computation
  - Function from Kelvin to Celsius: $x \mapsto x - 273.15$
  - Function from Celsius to Fahrenheit: $x \mapsto x \times \dfrac{9}{5} + 32$

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    - Warning! This is the reverse order of operations from the definition of relation composition

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  - Intensional view of function composition: $(f \cdot g)(x) = f(g(x))$
    - First apply g to x, then apply f to $g(x)$ (the output of g)
    - Right to left, or inside to outside

Functions

- The characteristic function of a set A is the function that maps all elements of A to **True** and all objects in the universe (or domain) U that are not elements of A to **False**
  - Relations, as sets, can be represented as characteristic functions

Functions

- Exercise from the book
  - **Exercise 2.10** The successor function $s : \mathbb{N} \to \mathbb{N}$ on the natural numbers is given by $n \mapsto n + 1$. What is the composition of s with itself?