Predicate Logic

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Announcements

- For 10/13
  - Read van Eijck and Unger Chapter 8
- For 10/19
  - HW2 due
  - Paper Presentation Ideas due

Today's Plan

- Paper Presentation Idea: Meaning Representations
- Predicate Logic
  - Syntax
  - Semantics
- User-defined Data Types

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- Paper Presentation Idea: Meaning Representations
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- (we'll see how far we get...)

Meaning Representations

- A few frameworks
  - Abstract Meaning Representation (AMR): Banarescu et al. 2013. Abstract Meaning Representation for Sembanking. Proceedings of LAW.
  - Uniform Meaning Representation (UMR): Van Gysel et al. 2021. Designing a Uniform Meaning Representataion for Natural Language Processing. Künstliche Intelligenz, 35:343–360.
  - Universal Conceptual Cognitive Annotation (UCCA): Abend and Rappoport. 2013. Universal Conceptual Cognitive Annotation (UCCA). Proceedings of ACL.
- And a corpus
  - Parallel Meaning Bank (PMB): Abzianidze et al. 2017. The Parallel Meaning Bank: Towards a Multilingual Corpus of Translations Annotated with Compositional Meaning Representations. Proceedings of EACL.
- Also see Designing Meaning Representations workshop

Predicate Logic

- "There are many aspects of natural language that propositional logic cannot express. For example, when translating the sentences (4.10) and (4.11) into propositional logic, the connection between their meanings is lost because they would have to be represented as completely different proposition constants p and q."
  - (4.10) Every prince saw a lady.
  - (4.11) Some prince saw a beautiful lady.

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  - (4.10) Every prince saw a lady.
  - (4.11) Some prince saw a beautiful lady.
- "Predicate logic is an extension of propositional logic with structured basic propositions and quantifications."

Predicate Logic

- Structured basic propositions
  - Instead of atomic propositions, consider predicates (relations) and variables/constants (individual entities)

Predicate Logic

- Structured basic propositions
  - Instead of atomic propositions, consider predicates (relations) and variables/constants (individual entities)
  - Predicates typically indicated by upper case letters P, Q, R, etc., possibly with indices
    - Also words like Prince, Saw, etc.
  - Variables typically indicated by lower case letters x, y, z, etc., possibly with indices
  - Constants typically indicated by lower case letters a, b, c, etc., possibly with indices
    - Usually it should be clear from context whether a letter is a variable or a constant

Predicate Logic

- Let v_1, v_2, and v_3 be variables or constants, P be a one-place predicate, R be a two-place predicate, S be a three-place predicate, etc. Then the following are formulas of predicate logic:
  - $P(v_1)$ (or Pv_1)
  - $R(v_1, v_2)$ (or Rv_1v_2)
  - $S(v_1, v_2, v_3)$ (or $Sv_1v_2v_3$)
  - etc.

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  - etc.

  - For example, let:
    - Prince(x) be "x is a prince"
    - Saw(x, y) be "x saw y"

Predicate Logic

- Quantification
  - Let v be a variable (not a constant), and F be a formula. Then the following are also formulas:
    - Universal quantification: $\forall v.F$ ("for all v, F")
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  - (Also function symbols, which we won't talk about here)
    - (Constants can be seen as zero-place functions)

Predicate Logic

- For example, the sentence “Every prince saw a lady” is ambiguous between two readings:
  - $\forall x(\text{Prince}(x) \rightarrow \exists y(\text{Lady}(y) \land \text{Saw}(x, y)))$
    - “It holds for every prince that he saw some lady”
  - $\exists y(\text{Lady}(y) \land \forall x(\text{Prince}(x) \rightarrow \text{Saw}(x, y)))$
    - “There is one specific lady that every prince saw”

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- The sentence "Some prince saw a beautiful lady" has one reading:
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    - "There is one specific lady that every prince saw"
- The sentence "Some prince saw a beautiful lady" has one reading:
  - $\exists x \exists y(\text{Prince}(x) \wedge \text{Lady}(y) \wedge \text{Beautiful}(y) \wedge \text{Saw}(x, y))$
- "Note that universally quantified statements are translated with $\rightarrow$ as main connective, while in the existentially quantified statements we made use of $\wedge$. Think about why this is so."

Predicate Logic

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  - "An occurrence of x is bound in F if there is some quantifier occurrence that binds it, it is free otherwise."
  - "A predicate logical formula is called open if it contains at least one variable occurrence which is free; it is called closed otherwise."
  - "A closed predicate logical formula is also called a predicate logical sentence."

Semantics of Predicate Logic

- A model M for a predicate logical language L contains:
  - A domain of discourse D, which is a non-empty set of individual entities
  - An interpretation function I, which maps the relation symbols of L (with arity n) to the sets of n-tuples for which the relations hold, and also maps constants to the entities they denote

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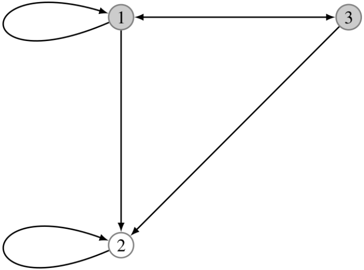
  - An interpretation function I, which maps the relation symbols of L (with arity n) to the sets of n-tuples for which the relations hold, and also maps constants to the entities they denote
- Also define a variable assignment or valuation function g that maps variables to entities
  - Let $g[v := d]$ be the valuation that is like g except for the fact that v gets value d

Semantics of Predicate Logic



- $D = \{1, 2, 3\}$
- $I(P) = \{1, 3\}$
- $I(R) = \{(1, 1), (1, 2), (1, 3), (2, 2), (3, 1), (3, 2)\}$

Semantics of Predicate Logic

- Given a model $M = (D, I)$ and a variable assignment g:
  - $P(v_1, ...)$ is true iff according to I, the entities denoted by $v_1, ...$ participate in the relation P
    - For variables v, look at $g(v)$; for constants c, look at $I(c)$
  - $v_1 = v_2$ is true iff v_1 and v_2 denote the same entity
  - $\forall v.F$ is true iff for all $d \in D$, F is true under the assignment $g[v := d]$
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- $M \models_g F$: F is true in M under assignment g

Semantics of Predicate Logic

- "If we evaluate closed formulas (formulas without free variables), the assignment g becomes irrelevant, so for a closed formula F we can simply put $M \models F$ iff there is some assignment g with $M \models_g F$."
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  - Assignments needed for recursive definition of truth
- "A predicate logical sentence F is called logically valid if F is true in every model."
  - Notation is $\models F$
- A predicate logical sentence C logically follows from sentences $P_1, \ldots, P_n$ if every model which makes $P_1, \ldots, P_n$ true also makes C true
  - Notation is $P_1, \ldots, P_n \models C$

Semantics of Predicate Logic

- **Exercise 5.17**
  1. Show that $\forall x(A(x) \wedge B(x))$ means something stronger than *All A are B*.
  2. Show that $\exists x(A(x) \rightarrow B(x))$ means something weaker than *Some A are B*.

Semantics of Predicate Logic

- **Exercise 5.18** Translate the following sentences into predicate logic, making sure that their truth conditions are captured.
  - Someone walks and someone talks.
  - No wizard cast a spell or mixed a potion.
  - Every balad that is sung by a princess is beautiful.
  - If a knight finds a dragon, he fights it.