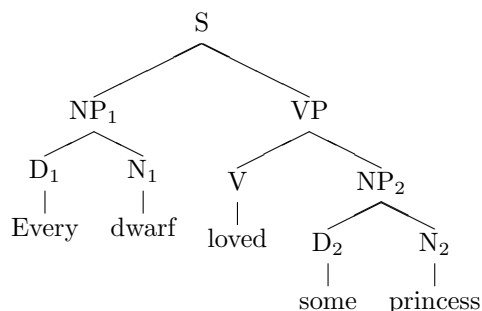


We want to compute the meaning of the sentence “Every dwarf loved some princess”. This sentence has the following syntactic structure:



We have the following basic (non-continuized) meanings:

$$\begin{aligned}
 \llbracket D_1 \rrbracket &= \llbracket \text{every} \rrbracket &= \lambda P \lambda Q. \forall x. P(x) \rightarrow Q(x) \\
 \llbracket N_1 \rrbracket &= \llbracket \text{dwarf} \rrbracket &= \lambda x. \text{Dwarf}(x) \text{ (or just Dwarf)} \\
 \llbracket V \rrbracket &= \llbracket \text{loved} \rrbracket &= \lambda y \lambda x. \text{Love}(x, y) \text{ (or just Love)} \\
 \llbracket D_2 \rrbracket &= \llbracket \text{some} \rrbracket &= \lambda P \lambda Q. \exists y. P(y) \wedge Q(y) \\
 \llbracket N_2 \rrbracket &= \llbracket \text{princess} \rrbracket &= \lambda x. \text{Princess}(x) \text{ (or just Princess)}
 \end{aligned}$$

In addition, we define the following functions:

$$\begin{aligned}
 \text{pure}(u) &= \lambda k. k(u)^1 \\
 u < * > v &= \lambda k. v(\lambda n. u(\lambda m. k(m(n))))^2 \\
 v > > = u &= \lambda c. v(\lambda a. (u(a))(c))
 \end{aligned}$$

We then have the following continuized grammar:

$$\begin{aligned}
 S \rightarrow NP VP & \quad \bar{S} = \overline{VP < * > NP} \\
 VP \rightarrow V NP & \quad \overline{VP} = \overline{V < * > NP} \\
 NP \rightarrow D N & \quad \overline{NP} = \overline{N > > = [D]} \\
 V \rightarrow \text{loved} & \quad \bar{V} = \text{pure}(\llbracket \text{loved} \rrbracket) \\
 N \rightarrow \text{dwarf} & \quad \bar{N} = \text{pure}(\llbracket \text{dwarf} \rrbracket) \\
 N \rightarrow \text{princess} & \quad \bar{N} = \text{pure}(\llbracket \text{princess} \rrbracket) \\
 D \rightarrow \text{every} & \quad \llbracket D \rrbracket = \llbracket \text{every} \rrbracket \\
 D \rightarrow \text{some} & \quad \llbracket D \rrbracket = \llbracket \text{some} \rrbracket
 \end{aligned}$$

<sup>1</sup>van Eijck and Unger call this `cpsConst`; one could also call this `return`

<sup>2</sup>van Eijck and Unger call this `cpsApply`

First, we have  $\overline{N_2} = \text{pure}(\llbracket \text{princess} \rrbracket) = \lambda k.k(\text{Princess})$ . Then we can compute  $\overline{NP_2} = \overline{N_2} \gg \llbracket D_2 \rrbracket$  as follows:

$$\begin{aligned}
\overline{NP_2} &= \overline{\text{some princess}} = \overline{N_2} \gg \llbracket D_2 \rrbracket \\
&= \lambda c.\overline{N_2}(\lambda a.(\llbracket D_2 \rrbracket(a))(c)) \\
&= \lambda c.(\lambda k.k(\text{Princess}))(\lambda a.(\llbracket D_2 \rrbracket(a))(c)) \\
&= \lambda c.(\lambda a.(\llbracket D_2 \rrbracket(a))(c))(\text{Princess}) \\
&= \lambda c.(\llbracket D_2 \rrbracket(\text{Princess}))(c) \\
&= \lambda c.((\lambda P\lambda Q.\exists y.P(y) \wedge Q(y))(\text{Princess}))(c) \\
&= \lambda c.(\lambda Q.\exists y.\text{Princess}(y) \wedge Q(y))(c) \\
&= \lambda c.\exists y.\text{Princess}(y) \wedge c(y)
\end{aligned}$$

Then, we have  $\overline{V} = \text{pure}(\llbracket \text{loved} \rrbracket) = \lambda n.n(\text{Love})$ . Then we can compute  $\overline{VP} = \overline{V} \langle * \rangle \overline{NP_2}$  as follows:

$$\begin{aligned}
\overline{VP} &= \overline{\text{loved some princess}} = \overline{V} \langle * \rangle \overline{NP_2} \\
&= \lambda k.\overline{NP_2}(\lambda n.\overline{V}(\lambda m.k(m(n)))) \\
&= \lambda k.(\lambda c.\exists y.\text{Princess}(y) \wedge c(y))(\lambda n.\overline{V}(\lambda m.k(m(n)))) \\
&= \lambda k.\exists y.\text{Princess}(y) \wedge (\lambda n.\overline{V}(\lambda m.k(m(n))))(y) \\
&= \lambda k.\exists y.\text{Princess}(y) \wedge \overline{V}(\lambda m.k(m(y))) \\
&= \lambda k.\exists y.\text{Princess}(y) \wedge (\lambda n.n(\text{Love}))(\lambda m.k(m(y))) \\
&= \lambda k.\exists y.\text{Princess}(y) \wedge (\lambda m.k(m(y)))(\text{Love}) \\
&= \lambda k.\exists y.\text{Princess}(y) \wedge k(\text{Love}(y))
\end{aligned}$$

Similarly to  $\overline{N_2}$ , we have  $\overline{N_1} = \text{pure}(\llbracket \text{dwarf} \rrbracket) = \lambda k.k(\text{Dwarf})$ . In the same way, we can compute  $\overline{NP_1} = \overline{N_1} \gg \llbracket D_1 \rrbracket$  as follows:

$$\begin{aligned}
\overline{NP_1} &= \overline{\text{every dwarf}} = \overline{N_1} \gg \llbracket D_1 \rrbracket \\
&= \lambda c.\overline{N_1}(\lambda a.(\llbracket D_1 \rrbracket(a))(c)) \\
&= \lambda c.(\lambda k.k(\text{Dwarf}))(\lambda a.(\llbracket D_1 \rrbracket(a))(c)) \\
&= \lambda c.(\lambda a.(\llbracket D_1 \rrbracket(a))(c))(\text{Dwarf}) \\
&= \lambda c.(\llbracket D_1 \rrbracket(\text{Dwarf}))(c) \\
&= \lambda c.((\lambda P\lambda Q.\forall x.P(x) \rightarrow Q(x))(\text{Dwarf}))(c) \\
&= \lambda c.(\lambda Q.\forall x.\text{Dwarf}(x) \rightarrow Q(x))(c) \\
&= \lambda c.\forall x.\text{Dwarf}(x) \rightarrow c(x)
\end{aligned}$$

Finally, we can compute  $\bar{S} = \overline{\bar{VP} \langle * \rangle \bar{NP}_1}$  as follows:

$$\begin{aligned}
\bar{S} &= \overline{\text{every dwarf loved some princess}} = \overline{\bar{VP}_1 \langle * \rangle \bar{NP}_1} \\
&= \lambda k. \overline{\bar{NP}_1} (\lambda n. \overline{\bar{VP}} (\lambda m. k(m(n)))) \\
&= \lambda k. (\lambda c. \forall x. \text{Dwarf}(x) \rightarrow c(x)) (\lambda n. \overline{\bar{VP}} (\lambda m. k(m(n)))) \\
&= \lambda k. \forall x. \text{Dwarf}(x) \rightarrow (\lambda n. \overline{\bar{VP}} (\lambda m. k(m(n))))(x) \\
&= \lambda k. \forall x. \text{Dwarf}(x) \rightarrow \overline{\bar{VP}} (\lambda m. k(m(x))) \\
&= \lambda k. \forall x. \text{Dwarf}(x) \rightarrow (\lambda c. \exists y. \text{Princess}(y) \wedge c(\text{Love}(y))) (\lambda m. k(m(x))) \\
&= \lambda k. \forall x. \text{Dwarf}(x) \rightarrow \exists y. \text{Princess}(y) \wedge (\lambda m. k(m(x))) (\text{Love}(y)) \\
&= \lambda k. \forall x. \text{Dwarf}(x) \rightarrow \exists y. \text{Princess}(y) \wedge k((\text{Love}(y))(x)) \\
&= \lambda k. \forall x. \text{Dwarf}(x) \rightarrow \exists y. \text{Princess}(y) \wedge k(\text{Love}(x, y))
\end{aligned}$$

To get the meaning of S, we apply  $\bar{S}$  to the trivial continuation  $\lambda x.x$ :

$$\begin{aligned}
\llbracket S \rrbracket &= \llbracket \text{every dwarf loved some princess} \rrbracket = \bar{S}(\lambda x.x) \\
&= (\lambda k. \forall x. \text{Dwarf}(x) \rightarrow \exists y. \text{Princess}(y) \wedge k(\text{Love}(x, y))) (\lambda x.x) \\
&= \forall x. \text{Dwarf}(x) \rightarrow \exists y. \text{Princess}(y) \wedge (\lambda x.x)(\text{Love}(x, y)) \\
&= \forall x. \text{Dwarf}(x) \rightarrow \exists y. \text{Princess}(y) \wedge \text{Love}(x, y)
\end{aligned}$$

Note that with an alternative definition of  $u \langle * \rangle v = \lambda k. u(\lambda m. v(\lambda n. k(m(n))))$ <sup>3</sup>, we can derive the reverse scope. The derivation is left as an exercise for the reader<sup>4</sup>.

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<sup>3</sup>van Eijck and Unger call this `cpsApply`'

<sup>4</sup>Specifically, Exercise 11.6