Computational Semantics
Day 1: Getting Started with Haskell + Inference Engine for NL

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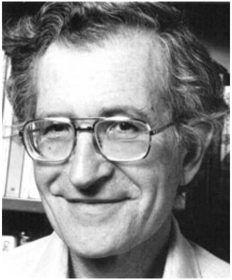
The formal study of natural language

1916 Ferdinand de Saussure proposes that natural language may be analyzed as a formal system.

1957 Noam Chomsky proposes to define natural languages as sets of grammatical sentences, and to study their structure with formal means.

- The ability of language users to recognize members of this set is called competence.
- Goal: Build a model of our linguistic knowledge, abstracting from language performance (speech disabilities, memory limitations, errors, etc). Such a model is called grammar.



1970 Richard Montague proposes to extend the Chomskyan program to semantics and pragmatics.

The birth of formal semantics

There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed, I consider it possible to comprehend the syntax and semantics of both kinds of languages within a single natural and mathematically precise theory.

(Richard Montague, 1930–1971)

In fact, when we descibe grammars of fragments of natural languages in a formal way, we are doing the same as when describing formal languages. (And this allows for a relatively straightforward implementation.)

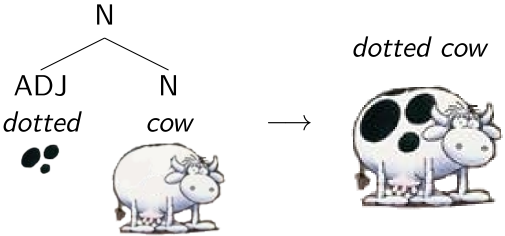
Organization of grammar

- **Phonology** investigates the smallest meaning-distinguishing units (speech sounds) and how they are combined into the smallest meaning-carrying units (morphemes).
- **Morphology** is concerned with how morphemes are combined into words.
- **Syntax** studies how words are combined into phrases and sentences.
- **Semantics** investigates the meanings of words, phrases and sentences, and how the meaning of a complex expression can be constructed from the meanings of its parts.

Our focus

We will concentrate on meaning and form at the level of phrases and sentences, i.e. start with words as basic building blocks.

Example:



A short history of Haskell

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In the 80s, efforts of researchers working on functional programming were scattered across many languages (Lisp, SASL, Miranda, ML,...).

In 1987 a dozen functional programmers decided to meet in order to reduce unnecessary diversity in functional programming languages by **designing a common language** that is

- based on ideas that enjoy a wide consensus
- suitable for further language research as well as applications, including building large systems
- freely available

A short history of Haskell

In 1990, they published the first **Haskell** specification, named after the logician and mathematician Haskell B. Curry (1900-1982).

Haskell is functional

A program consists entirely of functions.

- The main program itself is a function with the program's input as argument and the program's output as result.
- Typically the main function is defined in terms of other functions, which in turn are defined in terms of still more functions, until at the bottom level the functions are language primitives.

Running a Haskell program consists in evaluating expressions (basically functions applied to arguments).

A shift in thinking

Imperative thinking:

- Variables are pointers to storage locations whose value can be updated all the time.
- You give a sequence of commands telling the computer what to do step by step.

Examples:

- initialize a variable `examplelist` of type integer list, then add 1, then add 2, then add 3
- in order to compute the factorial of n, initialize an integer variable `f` as 1, then for all `i` from 1 to n, set `f` to `f`$\times$`i`

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Functional thinking:

- Variables are identifiers for an immutable, persistent value.
- You tell the computer what things are.

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```
factorial :: Int -> Int
factorial n = product [1..n]
```

A shift in thinking

Stop thinking in variable assignments, sequences and loops.

Start thinking in functions, immutable values and recursion.

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Haskell is rich enough to be useful. But above all, Haskell is a language in which people play. In the end, we want to infect your brain, not your hard drive.

(Simon Peyton-Jones)

Resources

- **For everything Haskell-related:** `haskell.org`.
- **Tutorials:**
  - Chapter 3 of our book
  - Real World Haskell
    `book.realworldhaskell.org/read/`
  - Learn you a Haskell for great good
    `learnyouahaskell.com`
  - A gentle introduction to Haskell
    `haskell.org/tutorial`

Getting started

Get the Haskell Platform:

- `http://hackage.haskell.org/platform/`

This includes the Glasgow Haskell Compiler (GHC) together with standard libraries and the interactive environment GHCi.

Haskell as a Calculator

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GHCi, version 6.12.3: http://www.haskell.org/ghc/  :? for help
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GHCi can be used to interactively evaluate expressions.

```
Prelude> 2 + 3
Prelude> 2 + 3 * 4
Prelude> 2^10
Prelude> (42 - 10) / 2
```

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3. Now you can evaluate expressions like `double 5`, `double (2+3)`, and `double (double 5)`.
4. With `:t` you can ask GHCi about the type of an expression.
5. Leave the interactive environment with `:q`.

Examples from Chapter 3 of the Book

```
module Day1

where

import Data.List
import Data.Char
```

Sentences can go on . . .

Sentences can go on

Sentences can go on . . .

Sentences can go on and on

Sentences can go on . . .

Sentences can go on and on and on

Sentences can go on . . .

Sentences can go on and on and on and on

Sentences can go on . . .

Sentences can go on and on and on and on and on

Sentences can go on . . .

Sentences can go on and on and on and on and on and on and on

Sentences can go on . . .

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```
gen :: Int -> String
gen 0 = "Sentences can go on"
gen n = gen (n-1) ++ " and on"

genS :: Int -> String
genS n = gen n ++ "."
```

A lazy list

```
sentences = "Sentences can go " ++ onAndOn

onAndOn = "on and " ++ onAndOn
```

Lambda Abstraction in Haskell

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- The result, the squared number, also has type `Int`.
- The function `sqr` is a function that, when combined with an argument of type `Int`, yields a value of type `Int`.
- This is precisely what the type-indication `Int -> Int` expresses.

String Functions in Haskell

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The types:

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Prelude> :t (\ x -> x ++ " emeritus")
\x -> x ++ " emeritus" :: [Char] -> [Char]
Prelude> :t "professor"
"professor" :: String
Prelude> :t (\ x -> x ++ " emeritus") "professor"
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The type indicates that (++) not only concatenates strings. It works for lists in general.

More String Functions in Haskell

```
Prelude> (\ x -> "nice " ++ x) "guy"
"nice guy"
Prelude> (\ f -> \ x -> "very " ++ (f x))
              (\ x -> "nice " ++ x) "guy"
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- Examples of characters are `'a'`, `'b'` (note the single quotes).
- Examples of strings are `"Montague"` and `"Chomsky"` (note the double quotes).
- In fact, `"Chomsky"` can be seen as an abbreviation of the following character list:

  `['C','h','o','m','s','k','y']`.

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- The head and tail are glued together by means of the operation `:`, of type `a -> [a] -> [a]`.
- The operation combines an object of type a with a list of objects of the same type to a new list of objects, again of the same type.

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List Reversal

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Reversal works for any list, not just for strings.

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To denote arbitrary types, Haskell allows the use of *type variables*. For these, `a`, `b`, . . . , are used.

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- By defining your own datatype from scratch, with a `data` type declaration. More about this in due course.

Mapping

If you use the Hugs command `:t` to find the types of the function `map`, you get the following:

```
Prelude> :t map
map :: forall a b. (a -> b) -> [a] -> [b]
```

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If you use the Hugs command `:t` to find the types of the function `map`, you get the following:

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Prelude> :t map
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The function `map` takes a function and a list and returns a list containing the results of applying the function to the individual list members.

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The function `map` takes a function and a list and returns a list containing the results of applying the function to the individual list members.

If `f` is a function of type `a -> b` and `xs` is a list of type `[a]`, then `map f xs` will return a list of type `[b]`. E.g., `map (^2) [1..9]` will produce the list of squares

```
[1, 4, 9, 16, 25, 36, 49, 64, 81]
```

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- Thus `(2^)` is the operation that computes powers of 2, and `map (2^) [1..10]` will yield

  `[2, 4, 8, 16, 32, 64, 128, 256, 512, 1024]`
- Similarly, `(>3)` denotes the property of being greater than 3, and `(3>)` the property of being smaller than 3.

Map

If *p* is a property (an operation of type `a -> Bool`) and `l` is a list of type `[a]`, then `map p l` will produce a list of type `Bool` (a list of truth values), like this:

```
Prelude> map (>3) [1..6]
[False, False, False, True, True, True]
Prelude>
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```
map :: (a -> b) -> [a] -> [b]
```

```
map f [] = []
map f (x:xs) = (f x) : map f xs
```

Filter

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```

```
filter :: (a -> Bool) -> [a] -> [a]
```

```
filter p [] = []
filter p (x:xs) | p x       = x : filter p xs
                | otherwise =     filter p xs
```

List comprehension

List comprehension is defining lists by the following method:

```
[ x | x <- xs, property x ]
```

This defines the sublist of `xs` of all items satisfying `property`. It is equivalent to:

```
filter property xs
```

Examples

```
someEvens     = [ x | x <- [1..1000], even x ]

evensUntil n = [ x | x <- [1..n], even x ]

allEvens     = [ x | x <- [1..], even x ]
```

Examples

```
someEvens     = [ x | x <- [1..1000], even x ]

evensUntil n = [ x | x <- [1..n], even x ]

allEvens      = [ x | x <- [1..], even x ]
```

Equivalently:

```
someEvens     = filter even [1..1000]

evensUntil n = filter even [1..n]

allEvens      = filter even [1..]
```

Nub

`nub` removes duplicates, as follows:

```
nub :: Eq a => [a] -> [a]
nub [] = []
nub (x:xs) = x : nub (filter (/= x) xs)
```

Function Composition

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(.) :: (a -> b) -> (c -> a) -> (c -> b)
f . g = \ x -> f (g x)
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- Note the types!

elem, all, and

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elem x (y:ys) = x == y || elem x ys
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all :: Eq a => (a -> Bool) -> [a] -> Bool
all p = and . map p
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Note the use of `.` for function composition.

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elem x (y:ys) = x == y || elem x ys
```

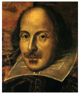
```
all :: Eq a => (a -> Bool) -> [a] -> Bool
all p = and . map p
```

Note the use of . for function composition.

```
and :: [Bool] -> Bool
and [] = True
and (x:xs) = x && and xs
```

Sonnet 73

```
sonnet73 =
 "That time of year thou mayst in me behold\n"
 ++ "When yellow leaves, or none, or few, do hang\n"
 ++ "Upon those boughs which shake against the cold,\n"
 ++ "Bare ruin'd choirs, where late the sweet birds sang.\n"
 ++ "In me thou seest the twilight of such day\n"
 ++ "As after sunset fadeth in the west,\n"
 ++ "Which by and by black night doth take away,\n"
 ++ "Death's second self, that seals up all in rest.\n"
 ++ "In me thou see'st the glowing of such fire\n"
 ++ "That on the ashes of his youth doth lie,\n"
 ++ "As the death-bed whereon it must expire\n"
 ++ "Consumed with that which it was nourish'd by.\n"
 ++ "This thou perceivest, which makes thy love more strong,\n"
 ++ "To love that well which thou must leave ere long."
```



Counting

```
count :: Eq a => a -> [a] -> Int
count x []                 = 0
count x (y:ys) | x == y    = succ (count x ys)
               | otherwise = count x ys
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               | otherwise = count x ys
```

```
average :: [Int] -> Rational
average [] = error "empty list"
average xs = toRational (sum xs) / toRational (length xs)
```

Some Commands to Try Out

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- `putStrLn sonnet73`

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- `putStrLn sonnet73`
- `map toLower sonnet73`

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- `putStrLn sonnet73`
- `map toLower sonnet73`
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- `count ’t’ sonnet73`
- `count ’t’ (map toLower sonnet73)`

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- `map toLower sonnet73`
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- `count ’t’ (map toLower sonnet73)`
- `count "thou" (words sonnet73)`

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Next, attempt exercise 3.16 on page 51 of the book.

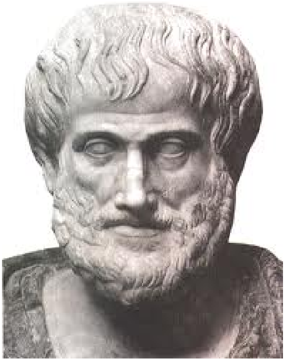
Example

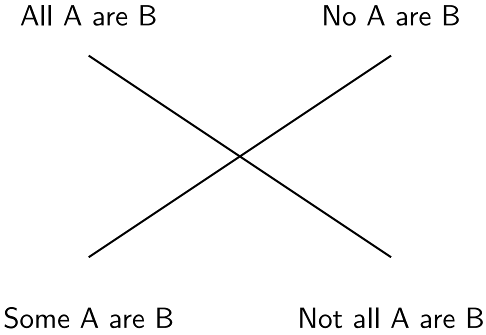
An inference engine with a natural language interface

Overview

- The Aristotelian quantifiers
- A natural language engine for talking about classes.
- Demo
- A tentative connection with cognitive realities.

The Aristotelian quantifiers





Aristotle interprets his quantifiers with existential import: *All A are B* and *No A are B* are taken to imply that there are *A*.

What can we ask or state with the Aristotelian quantifiers?

Questions and Statements (PN for plural nouns):

$$
\begin{array}{lll}
Q & ::= & \text{Are all PN PN?} \\
 & | & \text{Are no PN PN?} \\
 & | & \text{Are any PN PN?} \\
 & | & \text{Are any PN not PN?} \\
 & | & \text{What about PN?}
\end{array}
$$

$$
\begin{array}{lll}
S & ::= & \text{All PN are PN.} \\
 & | & \text{No PN are PN.} \\
 & | & \text{Some PN are PN.} \\
 & | & \text{Some PN are not PN.}
\end{array}
$$

Example interaction

```
user@home:~/courses/esslli2011$ ./Main
```

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user@home:~/courses/esslli2011$ ./Main
Welcome to the Knowledge Base.
Update or query the KB:
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user@home:~/courses/esslli2011$ ./Main
Welcome to the Knowledge Base.
Update or query the KB:
How about women?
```

Example interaction

```
user@home:~/courses/esslli2011$ ./Main
Welcome to the Knowledge Base.
Update or query the KB:
How about women?
All women are humans.
No women are men.

Update or query the KB:
```

Example interaction

```
user@home:~/courses/esslli2011$ ./Main
Welcome to the Knowledge Base.
Update or query the KB:
How about women?
All women are humans.
No women are men.

Update or query the KB:
All mammals are animals.
```

Example interaction

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user@home:~/courses/esslli2011$ ./Main
Welcome to the Knowledge Base.
Update or query the KB:
How about women?
All women are humans.
No women are men.

Update or query the KB:
All mammals are animals.
I knew that already.

Update or query the KB:
```

Example interaction

```
user@home:~/courses/esslli2011$ ./Main
Welcome to the Knowledge Base.
Update or query the KB:
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All women are humans.
No women are men.

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I knew that already.

Update or query the KB:
No mammals are birds.
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I knew that already.

Update or query the KB:
No mammals are birds.
OK.
```

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OK.

Update or query the KB:
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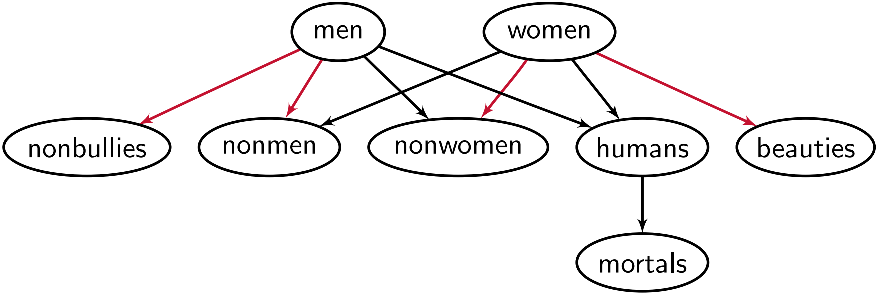
All women are humans.
All women are mammals.
No women are birds.
No women are men.
No women are owls.
```

Example knowledge base



The meanings of the Aristotelean quantifiers

- **ALL**: Set inclusion
- **SOME**: Non-empty set intersection
- **NOT ALL**: Non-inclusion
- **NO**: Empty intersection

Set inclusion:

- $A \subseteq B$ holds if and only if every element of A is an element of B.
- $A \not\subseteq B$ holds if and only if some element of A is not an element of B.

Complementation:

- Fix a universe U. $\overline{A} = U - A$ denotes the set of things in the universe that are not elements of A.

Implementation (in Haskell)

One possible implementation is given in Sections 4.3 and 5.7 of The Book.

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Here we present a different method, based on **reduction to propositional logic**.

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Here we present a different method, based on **reduction to propositional logic**.

Homework for you: compare the performance of the two versions.

Syllogistics and Propositional Logic

Key fact: A finite set of syllogistic forms Σ is unsatisable if and only if there exists an existential form ψ such that ψ taken together with the universal forms from Σ is unsatiable.

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This restricted form of satisability can easily be tested with propositional logic.

Talking about the Properties of a Single Object

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- An existential statement "Some A is B" gets translated as $a \wedge b$.
- For each property A we use a single proposition letter a.
- We have to check for *each* existential statement whether it is satisfiable when taken together with all universal statements.
- To test the satisfiability of a set of syllogistic statements with n existential statements we need n checks.

Literals, Clauses, Clause Sets

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Unit propagation for $\{q\}$ to this yields:

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yields

$$\{\{p\}, \{q\}, \{\neg q, r\}\}.$$

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- The *Horn fragment* of propositional logic consists of all clause sets where every clause has *at most one positive literal*.
- Satisfiability for syllogistic forms containing exactly one existental statement translates to the Horn fragment of propositional logic.
- HORNSAT is the problem of testing Horn clause sets for satisfiability.
- If unit propagation yields a clause set in which units $\{l\}, \{\bar{l}\}$ occur, the original clause set is unsatisfiable.
- Otherwise the units in the result determine a satisfying valuation.

HORNSAT

- The *Horn fragment* of propositional logic consists of all clause sets where every clause has *at most one positive literal*.
- Satisfiability for syllogistic forms containing exactly one existental statement translates to the Horn fragment of propositional logic.
- HORNSAT is the problem of testing Horn clause sets for satisfiability.
- If unit propagation yields a clause set in which units $\{l\}, \{\bar{l}\}$ occur, the original clause set is unsatisfiable.
- Otherwise the units in the result determine a satisfying valuation.
- Recipe: for all units $\{l\}$ occurring in the final clause set, map their proposition letter to the truth value that makes l true. Map all other proposition letters to false.

Module Declaration

```
module Syll where

import Data.List
import Data.Char
import System.IO
```

Literals, Clauses

```
data Lit = Pos String | Neg String deriving Eq

instance Show Lit where
  show (Pos x) = x
  show (Neg x) = '-': x

neg :: Lit -> Lit
neg (Pos x) = Neg x
neg (Neg x) = Pos x

type Clause = [Lit]

names :: [Clause] -> [String]
names = sort . nub . map nm . concat
  where nm (Pos x) = x
        nm (Neg x) = x
```

Unit Propagation (1)

```
unitProp :: Lit -> [Clause] -> [Clause]
unitProp x cs = concat (map (unitP x) cs)

unitP :: Lit -> Clause -> [Clause]
unitP x ys = if elem x ys then []
              else
               if elem (neg x) ys
                then [delete (neg x) ys]
                else [ys]

unit :: Clause -> Bool
unit [x] = True
unit  _  = False
```

Unit Propagation (2)

```
propagate :: [Clause] -> Maybe ([Lit],[Clause])
```

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```

```
propagate cls =
  prop [] (concat (filter unit cls)) (filter (not.unit) cls)
  where
    prop :: [Lit] -> [Lit] -> [Clause]
            -> Maybe ([Lit],[Clause])
    prop xs [] clauses = Just (xs,clauses)
    prop xs (y:ys) clauses =
      if elem (neg y) xs
       then Nothing
       else prop (y:xs)(ys++newlits) clauses' where
        newclauses = unitProp y clauses
        zs         = filter unit newclauses
        clauses'   = newclauses \\ zs
        newlits    = concat zs
```

KBs, Statements

```
type KB = ([Clause],[[Clause]])
-- first element: universal statements
-- second element: one clause list per existential statement

domain :: KB -> [Lit]
domain (xs,yss) =
   map (\ x -> Pos x) zs ++ map (\ x -> Neg x) zs
  where zs = names (xs ++ concat yss)

type Class = Lit

data Statement =
     All    Class    Class | No      Class  Class
   | Some   Class    Class | SomeNot Class  Class
   | AreAll Class Class    | AreNo   Class  Class
   | AreAny Class Class    | AnyNot  Class  Class
   | What   Class
  deriving Eq
```

Statement Display

```
instance Show Statement where
  show (All as bs)      =
    "All "  ++ show as ++ " are " ++ show bs ++ "."
  show (No as bs)       =
    "No "   ++ show as ++ " are " ++ show bs ++ "."
  show (Some as bs)     =
    "Some " ++ show as ++ " are " ++ show bs ++ "."
  show (SomeNot as bs) =
    "Some " ++ show as ++ " are not " ++ show bs ++ "."
  show (AreAll as bs)   =
    "Are all " ++ show as ++ show bs ++ "?"
  show (AreNo as bs)    =
    "Are no "  ++ show as ++ show bs ++ "?"
  show (AreAny as bs)   =
    "Are any " ++ show as ++ show bs ++ "?"
  show (AnyNot as bs)   =
    "Are any " ++ show as ++ " not " ++ show bs ++ "?"
  show (What as)        = "What about " ++ show as ++ "?"
```

Statement Classification, Query Negation

```
isQuery :: Statement -> Bool
isQuery (AreAll _ _)  = True
isQuery (AreNo _ _)   = True
isQuery (AreAny _ _)  = True
isQuery (AnyNot _ _)  = True
isQuery (What _)      = True
isQuery  _            = False

negat :: Statement -> Statement
negat (AreAll as bs) = AnyNot as bs
negat (AreNo  as bs) = AreAny as bs
negat (AreAny as bs) = AreNo  as bs
negat (AnyNot as bs) = AreAll as bs
```

The ⊂ Relation

```
subsetRel :: KB -> [(Class,Class)]
subsetRel kb =
  [(x,y) | x <- classes, y <- classes,
     propagate ([x]:[neg y]: fst kb) == Nothing ]
    where classes = domain kb
```

The ⊂ Relation

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```

If $R \subseteq A^2$ and $x \in A$, then $xR := \{y \mid (x, y) \in R\}$. This is called a *right section of a relation*.

```
rSection :: Eq a => a -> [(a,a)] -> [a]
rSection x r = [ y | (z,y) <- r, x == z ]
```

The $\subset$ Relation

```
subsetRel :: KB -> [(Class,Class)]
subsetRel kb =
  [(x,y) | x <- classes, y <- classes,
     propagate ([x]:[neg y]: fst kb) == Nothing ]
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If $R \subseteq A^2$ and $x \in A$, then $xR := \{y \mid (x, y) \in R\}$. This is called a *right section of a relation*.

```
rSection :: Eq a => a -> [(a,a)] -> [a]
rSection x r = [ y | (z,y) <- r, x == z ]
```

The supersets of a class are given by a right section of the subset relation. I.e. the supersets of a class are all classes of which it is a subset.

```
supersets :: Class -> KB -> [Class]
supersets cl kb = rSection cl (subsetRel kb)
```

The Relation of Having an Non-empty Intersection

```
intersectRel :: KB -> [(Class,Class)]
intersectRel kb@(xs,yys) =
 nub [(x,y) | x <- classes, y <- classes, lits <- litsList,
    elem x lits && elem y lits    ]
     where
       classes = domain kb
       litsList =
         [ maybe [] fst (propagate (ys++xs)) | ys <- yys ]
```

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       litsList =
         [ maybe [] fst (propagate (ys++xs)) | ys <- yys ]
```

```
intersectionsets :: Class -> KB -> [Class]
intersectionsets cl kb = rSection cl (intersectRel kb)
```

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Caution about KB Query

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- `derive kb stmt` is true. This means that the statement is derivable, hence true.
- `derive kb (neg stmt)` is true. This means that the negation of `stmt` is derivable, hence true. So `stmt` is false.
- neither `derive kb stmt` nor `derive kb (neg stmt)` is true. This means that the knowledge base has no information about `stmt`.

Derivability

```
derive :: KB -> Statement -> Bool
derive kb (AreAll as bs) = bs `elem` (supersets as kb)
derive kb (AreNo as bs)  = (neg bs) `elem` (supersets as kb)
derive kb (AreAny as bs) = bs `elem` (intersectionsets as kb)
derive kb (AnyNot as bs) = (neg bs) `elem`
                                (intersectionsets as kb)
```

Building a KB

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- If the update is successful, we want an updated knowledge base.
- If the update is not successful, we want to get an indication of failure.

Example: Update with an 'All' statement

The update function checks for possible inconsistencies. E.g., a request to add an $A \subseteq B$ fact to the knowledge base leads to an inconsistency if $A \not\subseteq B$ is already derivable.

```
update  :: Statement -> KB -> Maybe (KB,Bool)

update (All as bs) kb@(xs,yss)
  | bs' `elem` (intersectionsets as kb) = Nothing
  | bs `elem` (supersets  as kb) = Just (kb,False)
  | otherwise = Just (([as',bs]:xs,yss),True)
 where
   as' = neg as
   bs' = neg bs
```

Demo

...

Conclusions

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Conclusions

- Mini-case of computational semantics. What is the use of this?
- Cognitive research focusses on this kind of quantifier reasoning ...
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- The "natural logic for natural language" enterprise ...
- Towards Rational Reconstruction of Cognitive Processing