# <span id="page-0-0"></span>Compositional Distributional Models of Meaning

### Dimitri Kartsaklis Mehrnoosh Sadrzadeh

School of Electronic Engineering and Computer Science



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- Compositional distributional models of meaning (CDMs) extend distributional semantics to the phrase/sentence level.
- They provide a function that produces a vectorial representation of the meaning of a phrase or a sentence from the distributional vectors of its words.
- Useful in every NLP task: sentence similarity, paraphrase detection, sentiment analysis, machine translation etc.

#### In this tutorial:

We review three generic classes of CDMs: vector mixtures, tensor-based models and neural models.

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• How can we define Computational Linguistics?

Computational linguistics is the scientific and engineering discipline concerned with understanding written and spoken language from a computational perspective.

 $-$ Stanford Encyclopedia of Philosophy<sup>1</sup>

 $\frac{1}{1}$ http://plato.stanford.edu

### The principle of compositionality

The meaning of a complex expression is determined by the meanings of its parts and the rules used for combining them.

• Montague Grammar: A systematic way of processing fragments of the English language in order to get semantic representations capturing their meaning.

> There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians.

> > —Richard Montague, Universal Grammar (1970)

• A lexicon:

(1) a. every 
$$
\vdash
$$
 Dt :  $\lambda P.\lambda Q.\forall x[P(x) \rightarrow Q(x)]$   
b. man  $\vdash N : \lambda y.man(y)$ 

c. walks  $\vdash V_I : \lambda z.\mathit{walk}(z)$ 

A parse tree, so syntax guides the semantic composition:



$$
NP \rightarrow Dt \quad N: [\![N]\!]( [\![Dt]\!])
$$
  

$$
S \rightarrow NP \quad V_{IN}: [\![V_{IN}]\!]( [\![NP]\!])
$$

# Syntax-to-semantics correspondence (2/2)

• Logical forms of compounds are computed via  $\beta$ -reduction:



- The semantic value of a sentence can be true or false.
- **Q.** Can we do better than that?

#### Distributional hypothesis

Words that occur in similar contexts have similar meanings [Harris, 1958].

The functional interplay of philosophy and  $\frac{1}{2}$  should, as a minimum, guarantee... ...and among works of dystopian ? fiction...<br>The rapid advance in ? today su The rapid advance in 2 today suggests...<br>h are more popular in 2 -oriented schools ...calculus, which are more popular in 2 -oriented schools...<br>But because 2 is based on mathe But because ? is based on mathematics... ...the value of opinions formed in  $\cdot$  ? as well as in the religions... ...if  $\frac{?}{?}$  can discover the laws of human nature.... ...is an art, not an exact  $\frac{?}{?}$ ...factors shaping the future of our civilization: ? and religion. ...certainty which every new discovery in ? either replaces or reshapes. ...if the new technology of computer  $\begin{array}{cc} ? & \text{is to grow significantly} \\ \text{He got a} & ? & \text{scholarship to Yale} \end{array}$ He got a ? scholarship to Yale. ...frightened by the powers of destruction ? has given... ...but there is also specialization in  $\overline{\ }$  and technology...

#### Distributional hypothesis

Words that occur in similar contexts have similar meanings [Harris, 1958].



# Distributional models of meaning

• A word is a vector of co-occurrence statistics with every other word in a selected subset of the vocabulary:



• Semantic relatedness is usually based on cosine similarity:

$$
\text{sim}(\overrightarrow{v},\overrightarrow{u}) = \cos \theta_{\overrightarrow{v},\overrightarrow{u}} = \frac{\langle \overrightarrow{v} \cdot \overrightarrow{u} \rangle}{\|\overrightarrow{v}\| \|\overrightarrow{u}\|}
$$

## A real vector space



Distributional models of meaning are quantitative, but they do not scale up to phrases and sentences; there is not enough data:



Even if we had an infinitely large corpus, what the context of a sentence would be?

# The role of compositionality

#### Compositional distributional models

We can produce a sentence vector by composing the vectors of the words in that sentence.

$$
\overrightarrow{s} = f(\overrightarrow{w_1}, \overrightarrow{w_2}, \ldots, \overrightarrow{w_n})
$$

Three generic classes of CDMs:

- Vector mixture models Mitchell and Lapata (2010)]
- **Tensor-based models** Coecke, Sadrzadeh, Clark (2010); Baroni and Zamparelli (2010)]
- *Neural* models [Socher et al. (2012); Kalchbrenner et al. (2014)]



# Applications (1/2)

#### Why CDMs are important?

The problem of producing robust representations for the meaning of phrases and sentences is at the heart of every task related to natural language.

#### • Paraphrase detection

Problem: Given two sentences, decide if they say the same thing in different words

Solution: Measure the cosine similarity between the sentence vectors

#### • Sentiment analysis

Problem: Extract the general sentiment from a sentence or a document

Solution: Train a classifier using sentence vectors as input

### **• Textual entailment**

Problem: Decide if one sentence logically infers a different one Solution: Examine the feature inclusion properties of the sentence vectors

### **• Machine translation**

Problem: Automatically translate one sentence into a different language

Solution: Encode the source sentence into a vector, then use this vector to decode a surface form into the target language

• And so on. Many other potential applications exist...

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## Element-wise vector composition

• The easiest way to compose two vectors is by working element-wise [Mitchell and Lapata (2010)]:

$$
\overrightarrow{w_1w_2} = \alpha \overrightarrow{w_1} + \beta \overrightarrow{w_2} = \sum_i (\alpha c_i^{w_1} + \beta c_i^{w_2}) \overrightarrow{n_i}
$$

$$
\overrightarrow{w_1w_2} = \overrightarrow{w_1} \odot \overrightarrow{w_2} = \sum_i c_i^{w_1} c_i^{w_2} \overrightarrow{n_i}
$$

An element-wise "mixture" of the input elements:



- Words, phrases and sentences share the same vector space
- A bag-of-word approach. Word order does not play a role:

$$
\overrightarrow{dog} + \overrightarrow{bites} + \overrightarrow{man} = \overrightarrow{man} + \overrightarrow{bites} + \overrightarrow{dog}
$$

Feature-wise, vector addition can be seen as feature union, and vector multiplication as feature intersection

## Vector mixtures: Intuition

- The distributional vector of a word shows the extent to which this word is related to other words of the vocabulary
- For a verb, the components of its vector are related to the action described by the verb
- I.e. the vector for the word 'run' shows the extent to which a 'dog' can run, a 'car' can run, a 'table' can run and so on
- So, the element-wise composition of  $\overrightarrow{dog}$  with  $\overrightarrow{ruh}$  shows the extent to which things that are related to dogs can also run (and vice versa); in other words:

The resulting vector shows how compatible is the verb with the specific subject.

### Distinguishing feature:

All words contribute equally to the final result.

### PROS:

- Trivial to implement
- Surprisingly effective in practice

CONS:

- A bag-of-word approach
- Does not distinguish between the type-logical identities of the words

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## Relational words as functions

- In a vector mixture model, an adjective is of the same order as the noun it modifies, and both contribute equally to the result.
- One step further: Relational words are multi-linear maps (tensors of various orders) that can be applied to one or more arguments (vectors).



• Formalized in the context of compact closed categories by Coecke, Sadrzadeh and Clark (2010).

# Quantizing the grammar

### Coecke, Sadrzadeh and Clark (2010):

Pregroup grammars are structurally homomorphic with the category of finite-dimensional vector spaces and linear maps (both share compact closure)

• In abstract terms, there exists a structure-preserving passage from grammar to meaning:

 $\mathcal{F}:$  Grammar  $\rightarrow$  Meaning

• The meaning of a sentence  $w_1w_2 \ldots w_n$  with grammatical derivation  $\alpha$  is defined as:

$$
\overrightarrow{w_1w_2\ldots w_n}:=\mathcal{F}(\alpha)(\overrightarrow{w_1}\otimes\overrightarrow{w_2}\otimes\ldots\otimes\overrightarrow{w_n})
$$

A pregroup grammar  $P(\Sigma, \mathcal{B})$  is a relation that assigns grammatical types from a pregroup algebra freely generated over a set of atomic types  $\beta$  to words of a vocabulary  $\Sigma$ .

• A pregroup algebra is a partially ordered monoid, where each element  $p$  has a left and a right adjoint such that:

$$
\rho \cdot \rho^r \leq 1 \leq \rho^r \cdot \rho \qquad \quad \rho^l \cdot \rho \leq 1 \leq \rho \cdot \rho^l
$$

- Elements of the pregroup are basic (atomic) grammatical types, e.g.  $\mathcal{B} = \{n, s\}.$
- Atomic grammatical types can be combined to form types of higher order (e.g.  $n \cdot n'$  or  $n' \cdot s \cdot n'$ )
- A sentence  $w_1w_2 \ldots w_n$  (with word  $w_i$  to be of type  $t_i$ ) is grammatical whenever:

$$
t_1\cdot t_2\cdot\ldots\cdot t_n\leq s
$$

## Pregroup derivation: example



A monoidal category  $(C, \otimes, I)$  is compact closed when every object has a left and a right adjoint, for which the following morphisms exist:

$$
A \otimes A^r \xrightarrow{\epsilon^r} I \xrightarrow{\eta^r} A^r \otimes A \qquad A^l \otimes A \xrightarrow{\epsilon^l} I \xrightarrow{\eta^l} A \otimes A^l
$$

- Pregroup grammars are CCCs, with  $\epsilon$  and  $\eta$  maps corresponding to the partial orders
- FdVect, the category of finite-dimensional vector spaces and linear maps, is a also a (symmetric) CCC:
	- $\bullet$   $\epsilon$  maps correspond to inner product
	- $\bullet$   $\eta$  maps to identity maps and multiples of those

We define a strongly monoidal functor  $\mathcal F$  such that:

 $\mathcal{F}: P(\Sigma, \mathcal{B}) \to \mathsf{FdVect}$ 

$$
\mathcal{F}(p) = P \quad \forall p \in \mathcal{B}
$$
\n
$$
\mathcal{F}(1) = \mathbb{R}
$$
\n
$$
\mathcal{F}(p \cdot q) = \mathcal{F}(p) \otimes \mathcal{F}(q)
$$
\n
$$
\mathcal{F}(p') = \mathcal{F}(p') = \mathcal{F}(p)
$$
\n
$$
\mathcal{F}(p \leq q) = \mathcal{F}(p) \rightarrow \mathcal{F}(q)
$$
\n
$$
\mathcal{F}(\epsilon') = \mathcal{F}(\epsilon') = \text{inner product in \textbf{FdVect}}
$$
\n
$$
\mathcal{F}(\eta') = \mathcal{F}(\eta') = \text{identity maps in \textbf{FdVect}}
$$

# A multi-linear model

The grammatical type of a word defines the vector space in which the word lives:  $\bullet$  Nouns are vectors in  $N$ ; • adjectives are linear maps  $N \rightarrow N$ , i.e elements in  $N \otimes N$ : • intransitive verbs are linear maps  $N \rightarrow S$ , i.e. elements in  $N \otimes S$ ; • transitive verbs are bi-linear maps  $N \otimes N \rightarrow S$ , i.e. elements of  $N \otimes S \otimes N$ ;

• The composition operation is tensor contraction, i.e. elimination of matching dimensions by application of inner product.

## Categorical composition: example



## Creating relational tensors: Extensional approach

A relational word is defined as the set of its arguments:

 $\llbracket \text{red} \rrbracket = \{ \text{car}, \text{door}, \text{dress}, \text{ink}, \cdots \}$ 

• Grefenstette and Sadrzadeh (2011):

$$
\overline{adj} = \sum_i \overrightarrow{noun_i} \qquad \overrightarrow{verb_{int}} = \sum_i \overrightarrow{subj_i} \qquad \overrightarrow{verb_{tr}} = \sum_i \overrightarrow{subj_i} \otimes \overrightarrow{obj_i}
$$

• Kartsaklis and Sadrzadeh (2016):

$$
\overline{adj} = \sum_{i} \overrightarrow{noun_i} \otimes \overrightarrow{noun_i} \qquad \overrightarrow{verb_{int}} = \sum_{i} \overrightarrow{subj_i} \otimes \overrightarrow{subj_i}
$$
\n
$$
\overrightarrow{verb_{tr}} = \sum_{i} \overrightarrow{subj_i} \otimes \left( \frac{\overrightarrow{subj_i} + \overrightarrow{obj_i}}{2} \right) \otimes \overrightarrow{obj_i}
$$

# Creating relational tensors: Statistical approach

#### Baroni and Zamparelli (2010):

Create holistic distributional vectors for whole compounds (as if they were words) and use them to train a linear regression model.



- Certain classes of words, such as determiners, relative pronouns, prepositions, or coordinators occur in almost every possible context.
- Thus, they are considered semantically vacuous from a distributional perspective and most often they are simply ignored.

In the tensor-based setting, these special words can be modelled by exploiting additional mathematical structures, such as Frobenius algebras and bialgebras.

## Frobenius algebras in FdVect

 $\bullet$  Given a symmetric CCC ( $\mathcal{C}, \otimes, I$ ), an object  $X \in \mathcal{C}$  has a Frobenius structure on it if there exist morphisms:

 $\Delta: X \to X \otimes X$ ,  $\iota: X \to I$  and  $\mu: X \otimes X \to X$ ,  $\zeta: I \to X$ 

conforming to the Frobenius condition:

$$
(\mu \otimes 1_X) \circ (1_X \otimes \Delta) = \Delta \circ \mu = (1_X \otimes \mu) \circ (\Delta \otimes 1_X)
$$

In  $\mathsf{FdVect}$ , any vector space  $V$  with a fixed basis  $\{\overrightarrow{v_i}\}_i$  has a commutative special Frobenius algebra over it [Coecke and Pavlovic, 2006]:

$$
\Delta : \overrightarrow{v_i} \mapsto \overrightarrow{v_i} \otimes \overrightarrow{v_i} \qquad \qquad \mu : \overrightarrow{v_i} \otimes \overrightarrow{v_i} \mapsto \overrightarrow{v_i}
$$

• It can be seen as copying and merging of the basis.

## Frobenius algebras: Relative pronouns

How to represent relative pronouns in a tensor-based setting?

A relative clause modifies the head noun of a phrase:



• The result is a merging of the vectors of the noun and the relative clause:

$$
\overrightarrow{mah} \odot (\overrightarrow{likes} \times \overrightarrow{Mary})
$$

[Sadrzadeh, Clark, Coecke (2013)]

## Frobenius algebras: Coordination

Copying and merging are the key processes in coordination:



- The subject is copied by a ∆-map and interacts individually with the two verbs
- $\bullet$  The results are merged together with a  $\mu$ -map

$$
\overrightarrow{\textit{John}}^{\textsf{T}} \times (\overrightarrow{\textit{sleep}} \odot \overrightarrow{\textit{snore}})
$$

[Kartsaklis (2016)]

Tensor-based composition goes beyond a simple compatibility check between the two argument vectors; it transforms the input into an output of possibly different type.

A verb, for example, is a function that takes as input a noun and transforms it into a sentence:

$$
f_{int}: N \to S \qquad f_{tr}: N \times N \to S
$$

- Size and form of the sentence space become tunable parameters of the models, and can depend on the task.
- Taking  $S = \{\left(\begin{smallmatrix}0\ 1\end{smallmatrix}\right), \left(\begin{smallmatrix}1\ 0\end{smallmatrix}\right)\}$ , for example, provides an equivalent to formal semantics.

## Tensor-based models: Pros and Cons

### Distinguishing feature:

Relational words are multi-linear maps acting on arguments

PROS:

- Aligned with the formal semantics perspective
- More powerful than vector mixtures
- Flexible regarding the representation of functional words, such as relative pronouns and prepositions.

CONS:

- Every logical and functional word must be assigned to an appropriate tensor representation–it's not always clear how
- Space complexity problems for functions of higher arity (e.g. a ditransitive verb is a tensor of order 4)

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## An artificial neuron



- $\bullet$  The  $x_i$ s form the input vector
- $\bullet$  The  $w_{ij}$ s is a set of weights associated with the *i*-th output of the layer
- $\bullet$  f is a non-linear function such as tanh or sigmoid
- $a_i$  is the *i*-th output of the layer, computed as:

$$
a_i = f(w_{1i}x_1 + w_{2i}x_2 + w_{3i}x_3)
$$

# A simple neural net

A feed-forward neural network with one hidden layer:



- Note that  $\mathbf{W}^{(1)} \in \mathbb{R}^{3 \times 5}$  and  $\mathbf{W}^{(2)} \in \mathbb{R}^{2 \times 3}$
- $\bullet$  f is a non-linear function such as tanh or sigmoid (take  $f =$  Id and you have a tensor-based model)
- A universal approximator
- The goal of NN training is to find the set of parameters that optimizes a given objective function
- Or, to put it differently, to minimize an error function.
- Assume, for example, the goal of the NN is to produce a vector  $\overrightarrow{v}$  that matches a specific target vector  $\overrightarrow{t}$ . The function:

$$
E = \frac{1}{2m} \sum_i ||\overrightarrow{t_i} - \overrightarrow{y_i}||^2
$$

gives the total error across all training instances.

 $\bullet$  We want to set the weights of the NN such that  $E$  becomes zero or as close to zero as possible.

Take steps proportional to the *negative* of the gradient of E at the current point.



$$
\Theta_t = \Theta_{t-1} - \alpha \nabla E(\Theta_{t-1})
$$

- $\Theta_t$ : the parameters of the model at time step t
- $\bullet$   $\alpha$ : a learning rate

(Graph taken from "The Beginner Programmer" blog, http://firsttimeprogrammer.blogspot.co.uk)

## Backpropagation of errors

• How do we compute the error terms at the inner layers?

These are computed based one the errors of the next layer by using backpropagation. In general:

$$
\delta_k = \Theta_k^{\mathsf{T}} \delta_{k+1} \odot f'(z_k)
$$

- $\bullet$   $\delta_k$  is the error vector at layer k
- $\Theta_k$  is the weight matrix of layer k
- $\bullet$   $z_k$  is the weighted sum at the output of layer k
- $f'$  is the derivative of the non-linear function  $f$
- Just an application of the chain rule for derivatives.

## Recurrent and recursive NNs

- **•** Standard NNs assume that inputs are independent of each other
- That is not the case in language; a word, for example, always depends on the previous words in the same sentence
- In a recurrent NN, connections form a directed cycle so that each output depends on the previous ones
- A recursive NN is applied recursively following a specific structure.



## Recursive neural networks for composition



## Unsupervised learning with NNs

- $\bullet$  How can we train a NN in an unsupervised manner?
- Train the network to reproduce its input via an expansion layer:



Use the output of the hidden layer as a compressed version of the input [Socher et al. (2011)]

# Long Short-Term Memory networks (1/2)

• RNNs are effective, but fail to capture long-range dependencies such as:

### **The movie** I liked and John said Mary and Ann really hated

- "Vanishing gradient" problem: Back-propagating the error requires the multiplication of many very small numbers together, and training for the bottom layers starts to stall.
- Long Short-Term Memory networks (LSTMs) (Hochreiter and Schmidhuber, 1997) provide a solution, by equipping each neuron with an internal state.

# Long Short-Term Memory networks (2/2)



(Diagrams taken from Christopher Olah's blog, http://colah.github.io/)

NN-based methods come mainly from image processing. How can we make them more linguistically aware?

### Cheng and Kartsaklis (2015):

- **•** Take into account syntax, by optimizing against a scrambled version of each sentence
- **•** Dynamically disambiguate the meaning of words during training based on their main<br>CONTEXT main



# Convolutional NNs

- Originated in pattern recognition [Fukushima, 1980]
- Small filters apply on every position of the input vector:



- Capable of extracting fine-grained local features independently of the exact position in input
- **•** Features become increasingly global as more layers are stacked
- Each convolutional layer is usually followed by a pooling layer
- Top layer is fully connected, usually a soft-max classifier
- Application to language: Collobert and Weston (2008)

# DCNNs for modelling sentences



(Figures reused with permission)

# Beyond sentence level

An additional convolutional layer can provide document vectors [Denil et al. (2014)]:



- Recall that tensor-based composition involves a linear transformation of the input into some output.
- Neural models make this process more effective by applying consecutive non-linear layers of transformation.

A NN does not only project a noun vector onto a sentence space, but it can also transform the geometry of the space itself in order to make it reflect better the relationships between the points (sentences) in it.

# Neural models: Intuition (2/2)

**• Example:** Although there is no linear map to send an input  $x \in \{0, 1\}$  to the correct XOR value, the function can be approximated by a simple NN with one hidden layer.



• Points in (b) can be seen as representing two semantically distinct groups of sentences, which the NN is able to distinguish (while a linear map cannot)

# Neural models: Pros and Cons

### Distinguishing feature:

Drastic transformation of the sentence space.

PROS:

- Non-linearity and layered approach allow the simulation of a very wide range of functions
- Word vectors are parameters of the model, optimized during training
- State-of-the-art results in a number of NLP tasks

CONS:

- Requires expensive training based on backpropagation
- Difficult to discover the right configuration
- A "black-box" approach: not easy to correlate inner workings with output

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- No convincing solution for logical connectives, negation, quantifiers and so on.
- Functional words, such as prepositions and relative pronouns, are also a problem.
- Sentence space is usually identified with word space. This is convenient, but is it the right thing to do?
- Solutions depend on the specific CDM class—e.g. not much to do in a vector mixture setting
- **Important:** How can we make NNs more linguistically aware? [Cheng and Kartsaklis (2015)]
- CDMs provide quantitative semantic representations for sentences (or even documents)
- Element-wise operations on word vectors constitute an easy and reasonably effective way to get sentence vectors
- Categorical compositional distributional models allow reasoning on a theoretical level—a glass box approach
- Neural models are extremely powerful and effective; still a black-box approach, not easy to explain why a specific configuration works and some other does not.
- Convolutional networks seem to constitute the most promising solution to the problem of capturing the meaning of sentences

# Thank you for your attention!

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