Compositional Distributional Models of Meaning

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- Compositional distributional models of meaning (CDMs) extend distributional semantics to the phrase/sentence level.
- They provide a function that produces a vectorial representation of the meaning of a phrase or a sentence from the distributional vectors of its words.
- Useful in every NLP task: sentence similarity, paraphrase detection, sentiment analysis, machine translation etc.

In this tutorial:

We review three generic classes of CDMs: vector mixtures, tensor-based models and neural models.

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• How can we define Computational Linguistics?

Computational linguistics is the scientific and engineering discipline concerned with understanding written and spoken language from a computational perspective.

-Stanford Encyclopedia of Philosophy¹

¹http://plato.stanford.edu

The principle of compositionality

The meaning of a complex expression is determined by the meanings of its parts and the rules used for combining them.

• Montague Grammar: A systematic way of processing fragments of the English language in order to get semantic representations capturing their meaning.

There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians.

-Richard Montague, Universal Grammar (1970)

Syntax-to-semantics correspondence (1/2)

- A lexicon:
 - (1) a. every $\vdash Dt : \lambda P \cdot \lambda Q \cdot \forall x [P(x) \to Q(x)]$
 - b. man $\vdash N : \lambda y.man(y)$
 - c. walks $\vdash V_I : \lambda z. walk(z)$

• A parse tree, so syntax guides the semantic composition:



$$\begin{array}{ll} NP \rightarrow Dt & N : \llbracket N \rrbracket (\llbracket Dt \rrbracket) \\ S \rightarrow NP & V_{IN} : \llbracket V_{IN} \rrbracket (\llbracket NP \rrbracket) \end{array}$$

Syntax-to-semantics correspondence (2/2)

• Logical forms of compounds are computed via β -reduction:



- The semantic value of a sentence can be true or false.
- Can we do better than that?

Distributional hypothesis

Words that occur in similar contexts have similar meanings [Harris, 1958].

The functional interplay of philosophy and should, as a minimum, guarantee... ? ? ...and among works of dystopian fiction The rapid advance in ? today suggests... ? -oriented schools. ...calculus, which are more popular in But because ? is based on mathematics ? ... the value of opinions formed in as well as in the religions... ? can discover the laws of human nature....if ... is an art. not an exact ? ? ...factors shaping the future of our civilization: and religion. ...certainty which every new discovery in ? either replaces or reshapes. ? ... if the new technology of computer is to grow significantly ? He got a scholarship to Yale. ? ...frightened by the powers of destruction has given... ? ... but there is also specialization in and technology...

Distributional hypothesis

Words that occur in similar contexts have similar meanings [Harris, 1958].

science	should, as a minimum, guarantee
science	fiction
science	today suggests
science	-oriented schools.
science	is based on mathematics
science	as well as in the religions
science	can discover the laws of human nature
science	
science	and religion.
science	either replaces or reshapes.
science	is to grow significantly
science	scholarship to Yale.
science	has given
science	and technology
	science science science science science science science science science science science science

Distributional models of meaning

• A word is a vector of co-occurrence statistics with every other word in a selected subset of the vocabulary:



• Semantic relatedness is usually based on cosine similarity:

$$sim(\overrightarrow{v},\overrightarrow{u}) = \cos\theta_{\overrightarrow{v},\overrightarrow{u}} = \frac{\langle \overrightarrow{v}\cdot\overrightarrow{u}\rangle}{\|\overrightarrow{v}\|\|\overrightarrow{u}\|}$$

A real vector space



The necessity for a unified model

• Distributional models of meaning are quantitative, but they do not scale up to phrases and sentences; there is not enough data:

e	"natural language has great expressive power"								
	Web	News	Videos	Images	Shopping	More -	Sea		
	1 result	(0.33 seco							

Even if we had an infinitely large corpus, what the context of a sentence would be?

The role of compositionality

Compositional distributional models

We can produce a sentence vector by composing the vectors of the words in that sentence.

$$\overrightarrow{s} = f(\overrightarrow{w_1}, \overrightarrow{w_2}, \dots, \overrightarrow{w_n})$$

Three generic classes of CDMs:

- Vector mixture models [Mitchell and Lapata (2010)]
- *Tensor-based* models [Coecke, Sadrzadeh, Clark (2010); Baroni and Zamparelli (2010)]
- Neural models [Socher et al. (2012); Kalchbrenner et al. (2014)]



Applications (1/2)

Why CDMs are important?

The problem of producing robust representations for the meaning of phrases and sentences is at the heart of every task related to natural language.

• Paraphrase detection

Problem: Given two sentences, decide if they say the same thing in different words

Solution: Measure the cosine similarity between the sentence vectors

Sentiment analysis

Problem: Extract the general sentiment from a sentence or a document

Solution: Train a classifier using sentence vectors as input

Textual entailment

Problem: Decide if one sentence logically infers a different one Solution: Examine the feature inclusion properties of the sentence vectors

Machine translation

Problem: Automatically translate one sentence into a different language

Solution: Encode the source sentence into a vector, then use this vector to decode a surface form into the target language

• And so on. Many other potential applications exist...

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Element-wise vector composition

• The easiest way to compose two vectors is by working element-wise [Mitchell and Lapata (2010)]:

$$\overrightarrow{w_1w_2} = \alpha \overrightarrow{w_1} + \beta \overrightarrow{w_2} = \sum_i (\alpha c_i^{w_1} + \beta c_i^{w_2}) \overrightarrow{n_i}$$
$$\overrightarrow{w_1w_2} = \overrightarrow{w_1} \odot \overrightarrow{w_2} = \sum_i c_i^{w_1} c_i^{w_2} \overrightarrow{n_i}$$

• An element-wise "mixture" of the input elements:



- Words, phrases and sentences share the same vector space
- A bag-of-word approach. Word order does not play a role:

$$\overrightarrow{dog} + \overrightarrow{bites} + \overrightarrow{mah} = \overrightarrow{mah} + \overrightarrow{bites} + \overrightarrow{dog}$$

 Feature-wise, vector addition can be seen as feature union, and vector multiplication as feature intersection

Vector mixtures: Intuition

- The distributional vector of a word shows the extent to which this word is related to other words of the vocabulary
- For a verb, the components of its vector are related to the action described by the verb
- I.e. the vector for the word 'run' shows the extent to which a 'dog' can run, a 'car' can run, a 'table' can run and so on
- So, the element-wise composition of dog with run shows the extent to which things that are related to dogs can also run (and vice versa); in other words:

The resulting vector shows how compatible is the verb with the specific subject.

Distinguishing feature:

All words contribute equally to the final result.

PROS:

- Trivial to implement
- Surprisingly effective in practice

CONS:

- A bag-of-word approach
- Does not distinguish between the type-logical identities of the words

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Relational words as functions

- In a vector mixture model, an adjective is of the same order as the noun it modifies, and both contribute equally to the result.
- One step further: Relational words are multi-linear maps (tensors of various orders) that can be applied to one or more arguments (vectors).



• Formalized in the context of compact closed categories by Coecke, Sadrzadeh and Clark (2010).

Quantizing the grammar

Coecke, Sadrzadeh and Clark (2010):

Pregroup grammars are structurally homomorphic with the category of finite-dimensional vector spaces and linear maps (both share compact closure)

• In abstract terms, there exists a structure-preserving passage from grammar to meaning:

 $\mathcal{F}:\mathsf{Grammar}\to\mathsf{Meaning}$

The meaning of a sentence w₁w₂...w_n with grammatical derivation α is defined as:

$$\overrightarrow{w_1w_2\ldots w_n} := \mathcal{F}(\alpha)(\overrightarrow{w_1} \otimes \overrightarrow{w_2} \otimes \ldots \otimes \overrightarrow{w_n})$$

A pregroup grammar $P(\Sigma, B)$ is a relation that assigns grammatical types from a pregroup algebra freely generated over a set of atomic types B to words of a vocabulary Σ .

• A pregroup algebra is a partially ordered monoid, where each element *p* has a left and a right adjoint such that:

$$p \cdot p^r \leq 1 \leq p^r \cdot p \qquad p^l \cdot p \leq 1 \leq p \cdot p^l$$

- Elements of the pregroup are basic (atomic) grammatical types, e.g. B = {n, s}.
- Atomic grammatical types can be combined to form types of higher order (e.g. n · n^l or n^r · s · n^l)
- A sentence $w_1 w_2 \dots w_n$ (with word w_i to be of type t_i) is grammatical whenever:

$$t_1 \cdot t_2 \cdot \ldots \cdot t_n \leq s$$

Pregroup derivation: example



 A monoidal category (C, ⊗, I) is compact closed when every object has a left and a right adjoint, for which the following morphisms exist:

$$A \otimes A^r \xrightarrow{\epsilon^r} I \xrightarrow{\eta^r} A^r \otimes A \qquad \qquad A^I \otimes A \xrightarrow{\epsilon^I} I \xrightarrow{\eta^I} A \otimes A^I$$

- Pregroup grammars are CCCs, with ϵ and η maps corresponding to the partial orders
- **FdVect**, the category of finite-dimensional vector spaces and linear maps, is a also a (symmetric) CCC:
 - $\bullet~\epsilon$ maps correspond to inner product
 - η maps to identity maps and multiples of those

We define a strongly monoidal functor ${\mathcal F}$ such that:

 $\mathcal{F}: \textit{P}(\Sigma, \mathcal{B}) \to \textbf{FdVect}$

$$\begin{array}{rcl} \mathcal{F}(p) &=& P \quad \forall p \in \mathcal{B} \\ \mathcal{F}(1) &=& \mathbb{R} \\ \mathcal{F}(p \cdot q) &=& \mathcal{F}(p) \otimes \mathcal{F}(q) \\ \mathcal{F}(p^r) = \mathcal{F}(p^l) &=& \mathcal{F}(p) \\ \mathcal{F}(p \leq q) &=& \mathcal{F}(p) \rightarrow \mathcal{F}(q) \\ \mathcal{F}(\epsilon^r) = \mathcal{F}(\epsilon^l) &=& \text{inner product in FdVect} \\ \mathcal{F}(\eta^r) = \mathcal{F}(\eta^l) &=& \text{identity maps in FdVect} \end{array}$$

A multi-linear model

The grammatical type of a word defines the vector space in which the word lives:

- Nouns are vectors in N;
- adjectives are linear maps $N \rightarrow N$, i.e elements in $N \otimes N$;
- intransitive verbs are linear maps N → S, i.e. elements in N ⊗ S;

 transitive verbs are bi-linear maps N ⊗ N → S, i.e. elements of N ⊗ S ⊗ N;

 The composition operation is tensor contraction, i.e. elimination of matching dimensions by application of inner product.

Categorical composition: example



Creating relational tensors: Extensional approach

A relational word is defined as the set of its arguments:

$$\llbracket red \rrbracket = \{ car, door, dress, ink, \cdots \}$$

• Grefenstette and Sadrzadeh (2011):

$$\overrightarrow{adj} = \sum_{i} \overrightarrow{nouh}_{i} \qquad \overrightarrow{verb}_{int} = \sum_{i} \overrightarrow{subj}_{i} \qquad \overrightarrow{verb}_{tr} = \sum_{i} \overrightarrow{subj}_{i} \otimes \overrightarrow{obj}_{i}$$

• Kartsaklis and Sadrzadeh (2016):

$$\overline{adj} = \sum_{i} \overrightarrow{noun_{i}} \otimes \overrightarrow{noun_{i}} \qquad \overline{verb}_{int} = \sum_{i} \overrightarrow{subj_{i}} \otimes \overrightarrow{subj_{i}}$$
$$\overline{verb}_{tr} = \sum_{i} \overrightarrow{subj_{i}} \otimes \left(\frac{\overrightarrow{subj_{i}} + \overrightarrow{obj_{i}}}{2} \right) \otimes \overrightarrow{obj_{i}}$$

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Creating relational tensors: Statistical approach

Baroni and Zamparelli (2010):

Create holistic distributional vectors for whole compounds (as if they were words) and use them to train a linear regression model.



- Certain classes of words, such as determiners, relative pronouns, prepositions, or coordinators occur in almost every possible context.
- Thus, they are considered semantically vacuous from a distributional perspective and most often they are simply ignored.

In the tensor-based setting, these special words can be modelled by exploiting additional mathematical structures, such as Frobenius algebras and bialgebras.

Frobenius algebras in **FdVect**

 Given a symmetric CCC (C, ⊗, I), an object X ∈ C has a Frobenius structure on it if there exist morphisms:

 $\Delta: X \to X \otimes X \ , \ \iota: X \to I \qquad \text{and} \qquad \mu: X \otimes X \to X \ , \ \zeta: I \to X$

conforming to the Frobenius condition:

$$(\mu\otimes 1_X)\circ (1_X\otimes \Delta)=\Delta\circ \mu=(1_X\otimes \mu)\circ (\Delta\otimes 1_X)$$

In FdVect, any vector space V with a fixed basis { v_i}; has a commutative special Frobenius algebra over it [Coecke and Pavlovic, 2006]:

$$\Delta: \overrightarrow{v_i} \mapsto \overrightarrow{v_i} \otimes \overrightarrow{v_i} \qquad \mu: \overrightarrow{v_i} \otimes \overrightarrow{v_i} \mapsto \overrightarrow{v_i}$$

• It can be seen as copying and merging of the basis.

Frobenius algebras: Relative pronouns

How to represent relative pronouns in a tensor-based setting?

• A relative clause modifies the head noun of a phrase:



 The result is a merging of the vectors of the noun and the relative clause:

$$\overrightarrow{mah} \odot (\overrightarrow{likes} \times \overrightarrow{Mary})$$

[Sadrzadeh, Clark, Coecke (2013)]

Frobenius algebras: Coordination

Copying and merging are the key processes in coordination:



- The subject is copied by a Δ-map and interacts individually with the two verbs
- The results are merged together with a μ -map

$$\overrightarrow{\mathsf{John}}^\mathsf{T} \times (\overrightarrow{\mathsf{sleep}} \odot \overrightarrow{\mathsf{snore}})$$

[Kartsaklis (2016)]

Tensor-based composition goes beyond a simple compatibility check between the two argument vectors; it *transforms* the input into an output of possibly different type.

• A verb, for example, is a function that takes as input a noun and transforms it into a sentence:

$$f_{int}: N \to S$$
 $f_{tr}: N \times N \to S$

- Size and form of the sentence space become tunable parameters of the models, and can depend on the task.
- Taking S = { (⁰₁) , (¹₀) }, for example, provides an equivalent to formal semantics.

Tensor-based models: Pros and Cons

Distinguishing feature:

Relational words are multi-linear maps acting on arguments

PROS:

- Aligned with the formal semantics perspective
- More powerful than vector mixtures
- Flexible regarding the representation of functional words, such as relative pronouns and prepositions.

CONS:

- *Every* logical and functional word must be assigned to an appropriate tensor representation-it's not always clear how
- Space complexity problems for functions of higher arity (e.g. a ditransitive verb is a tensor of order 4)

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An artificial neuron



- The x_is form the input vector
- The *w_{ji}*s is a set of weights associated with the *i*-th output of the layer
- f is a non-linear function such as tanh or sigmoid
- *a_i* is the *i*-th output of the layer, computed as:

$$a_i = f(w_{1i}x_1 + w_{2i}x_2 + w_{3i}x_3)$$

A simple neural net

• A feed-forward neural network with one hidden layer:



- Note that $\mathbf{W}^{(1)} \in \mathbb{R}^{3 imes 5}$ and $\mathbf{W}^{(2)} \in \mathbb{R}^{2 imes 3}$
- f is a non-linear function such as tanh or sigmoid (take f = Id and you have a tensor-based model)
- A universal approximator

- The goal of NN training is to find the set of parameters that optimizes a given objective function
- Or, to put it differently, to minimize an error function.
- Assume, for example, the goal of the NN is to produce a vector \overrightarrow{y} that matches a specific target vector \overrightarrow{t} . The function:

$$E = \frac{1}{2m} \sum_{i} ||\overrightarrow{t_i} - \overrightarrow{y_i}||^2$$

gives the total error across all training instances.

• We want to set the weights of the NN such that *E* becomes zero or as close to zero as possible.

Take steps proportional to the *negative* of the gradient of E at the current point.



$$\Theta_t = \Theta_{t-1} - \alpha \nabla E(\Theta_{t-1})$$

- Θ_t: the parameters of the model at time step t
- α : a learning rate

(Graph taken from "The Beginner Programmer" blog, http://firsttimeprogrammer.blogspot.co.uk)

Backpropagation of errors

• How do we compute the error terms at the inner layers?

These are computed based one the errors of the next layer by using backpropagation. In general:

$$\delta_k = \Theta_k^\mathsf{T} \delta_{k+1} \odot f'(z_k)$$

- δ_k is the error vector at layer k
- Θ_k is the weight matrix of layer k
- z_k is the weighted sum at the output of layer k
- f' is the derivative of the non-linear function f
- Just an application of the chain rule for derivatives.

Recurrent and recursive NNs

- Standard NNs assume that inputs are independent of each other
- That is not the case in language; a word, for example, always depends on the previous words in the same sentence
- In a recurrent NN, connections form a directed cycle so that each output depends on the previous ones
- A recursive NN is applied recursively following a specific structure.



Recursive neural networks for composition



Unsupervised learning with NNs

- How can we train a NN in an unsupervised manner?
- Train the network to reproduce its input via an expansion layer:



• Use the output of the hidden layer as a compressed version of the input [Socher et al. (2011)]

Long Short-Term Memory networks (1/2)

• RNNs are effective, but fail to capture long-range dependencies such as:

The movie I liked and John said Mary and Ann really hated

- "Vanishing gradient" problem: Back-propagating the error requires the multiplication of many very small numbers together, and training for the bottom layers starts to stall.
- Long Short-Term Memory networks (LSTMs) (Hochreiter and Schmidhuber, 1997) provide a solution, by equipping each neuron with an internal state.

Long Short-Term Memory networks (2/2)



(Diagrams taken from Christopher Olah's blog, http://colah.github.io/)

NN-based methods come mainly from image processing. How can we make them more linguistically aware?

Cheng and Kartsaklis (2015):

- Take into account syntax, by optimizing against a scrambled version of each sentence
- Dynamically disambiguate the meaning of words during training based on their context



Convolutional NNs

- Originated in pattern recognition [Fukushima, 1980]
- Small filters apply on every position of the input vector:



- Capable of extracting fine-grained local features independently of the exact position in input
- Features become increasingly global as more layers are stacked
- Each convolutional layer is usually followed by a pooling layer
- Top layer is fully connected, usually a soft-max classifier
- Application to language: Collobert and Weston (2008)

DCNNs for modelling sentences



(Figures reused with permission)

Beyond sentence level

An additional convolutional layer can provide document vectors [Denil et al. (2014)]:



- Recall that tensor-based composition involves a linear transformation of the input into some output.
- Neural models make this process more effective by applying consecutive non-linear layers of transformation.

A NN does not only project a noun vector onto a sentence space, but it can also transform the geometry of the space itself in order to make it reflect better the relationships between the points (sentences) in it.

Neural models: Intuition (2/2)

Example: Although there is no linear map to send an input x ∈ {0,1} to the correct XOR value, the function can be approximated by a simple NN with one hidden layer.



• Points in (b) can be seen as representing two semantically distinct groups of sentences, which the NN is able to distinguish (while a linear map cannot)

Neural models: Pros and Cons

Distinguishing feature:

Drastic transformation of the sentence space.

PROS:

- Non-linearity and layered approach allow the simulation of a very wide range of functions
- Word vectors are parameters of the model, optimized during training
- State-of-the-art results in a number of NLP tasks

CONS:

- Requires expensive training based on backpropagation
- Difficult to discover the right configuration
- A "black-box" approach: not easy to correlate inner workings with output

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- No convincing solution for logical connectives, negation, quantifiers and so on.
- Functional words, such as prepositions and relative pronouns, are also a problem.
- Sentence space is usually identified with word space. This is convenient, but is it the right thing to do?
- Solutions depend on the specific CDM class—e.g. not much to do in a vector mixture setting
- Important: How can we make NNs more linguistically aware? [Cheng and Kartsaklis (2015)]

- CDMs provide quantitative semantic representations for sentences (or even documents)
- Element-wise operations on word vectors constitute an easy and reasonably effective way to get sentence vectors
- Categorical compositional distributional models allow reasoning on a theoretical level—a glass box approach
- Neural models are extremely powerful and effective; still a black-box approach, not easy to explain why a specific configuration works and some other does not.
- Convolutional networks seem to constitute the most promising solution to the problem of capturing the meaning of sentences

Thank you for your attention!

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